

Continuous Multi-Objective U-Turning Ant Colony Optimization with Partial-Decision Credit Assignment

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Abstract: This paper proposes a continuous multi-objective U-Turning Ant Colony Optimization (MO-U-TACO) method and evaluates its parameter configuration on 21 standard benchmarks from the ZDT, DTLZ, and WFG families. The original U-TACO concept assigns a preparatory role to partial U-ant routes before complete ants construct final tours. The present formulation transfers this distinction to continuous vectors: a U-ant samples a subset of coordinates, the remaining coordinates are completed, the full objective vector is evaluated, and pheromone credit is returned only to the sampled coordinates. A bounded external non-dominated archive, crowding-distance truncation, archive-guided interval sampling, and sparse local refinement replace the single incumbent used in single-objective applications. Researchers evaluated eight-parameter scenarios with 50 matched seeds per benchmark, producing 8,400 independent problem runs. Researchers retained the first 10 and 30 seeds as nested samples for run-count analysis. Researchers assessed performance using IGD+, normalized hypervolume, additive epsilon, archive size, function evaluations, and validation runtime. Under the 50-run analysis, S7 obtained the best mean composite rank (1.381) and the best rank on 12 of 21 problems. S8 ranked second (2.825) and produced the smallest arithmetic mean IGD+ (0.473454), but S7 achieved a larger mean hypervolume ratio (0.579963 versus 0.527794). The Friedman test detected a scenario effect ($\chi^2_F = 284.226667$, $p = 1.408e - 57$, Kendall's $W = 0.812076$). Holm-adjusted paired tests favoured S7 over every alternative, including S8 ($p_{Holm} = 2.931e - 14$). The ordering was stable across 10, 30, and 50 runs. These results identify the large-colony configuration as the most consistent setting under the adopted low-budget protocol; they do not compare it with established multi-objective algorithms.

Keywords: Multi-Objective Optimization; U-Turning Ant Colony Optimization; Pareto Archive; Hypervolume and Parameter Sensitivity; Run-Count Analysis; Validation Runtime.

Received on: 20/10/2025, **Revised on:** 13/12/2025, **Accepted on:** 04/02/2026, **Published on:** 03/06/2026

Journal Homepage: <https://www.fmdbpublish.com/user/journals/details/FTSCIS>

DOI: <https://doi.org/10.69888/FTSCIS.2026.000766>

Cite as: S. M. Almufti, R. R. Asaad, A. A. L. Ajibola, E. Aziz, A. Sherif, and A. Hassan, "Continuous Multi-Objective U-Turning Ant Colony Optimization with Partial-Decision Credit Assignment," *FMDB Transactions on Sustainable Critical Infrastructures*, vol. 1, no. 2, pp. 108–125, 2026.

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1. Introduction

Multi-objective optimization seeks decision vectors that balance two or more conflicting criteria. Unlike scalar optimization, the result is typically a set of mutually non-dominated alternatives rather than a single universally best solution [24]. An algorithm must therefore reduce the distance to the Pareto-optimal set while preserving a useful distribution of solutions across the objective space [8]; [10]. This dual requirement makes algorithm assessment sensitive to the benchmarks, reference sets, quality indicators, and statistical protocol selected. Ant Colony Optimization is a constructive population method in which pheromone accumulates on components associated with good solutions [7]. U-Turning Ant Colony Optimization introduced a second agent type, the U-ant, that traverses only part of a route and returns before a complete tour is formed [1]; [2]. The partial route is not treated as the final solution. Instead, it prepares pheromone information for ordinary ants, which then construct complete routes. Later comparisons reported that this additional guidance can improve route quality at the cost of additional function evaluations and runtime [3]. The same conceptual distinction applies outside routing if partial decisions are evaluated only after they are embedded in a complete candidate [6].

Continuous multi-objective search introduces three additional requirements. First, pheromone must describe intervals of real-valued coordinates rather than graph edges. Second, replace one incumbent with an archive of non-dominated solutions. Third, partial credit must account for both convergence and diversity. The method developed here addresses these requirements through interval pheromone, archive-guided completion, Pareto filtering, and crowding-distance control. The method is not a direct extension of the Great Deluge component used in the original traveling-salesman implementation; the continuous local search is feasibility-free because the selected benchmark suite is bounded and unconstrained. The study has four objectives. It defines a continuous multi-objective interpretation of U-TACO, preserves the distinction between partial guidance and complete solution construction, examines eight controlled parameter scenarios, and reports repeated-run evidence across benchmark families with different front shapes and search difficulties. The evaluation uses matched seeds and nested 10-, 30-, and 50-run samples. The analysis emphasizes parameter sensitivity within MO-U-TACO. No external-algorithm superiority claim is made because NSGA-II, SPEA2, MOEA/D, or other reference solvers were not rerun under the same budget [25].

2. Related Work

2.1. Multi-Objective Metaheuristics

Dominance-based methods such as NSGA-II use non-dominated sorting and density estimation [11]. SPEA2 combines dominance strength with an external archive [12]. MOEA/D decomposes the objective vector into a set of coordinated scalar subproblems [13], whereas NSGA-III uses reference directions to control diversity as the number of objectives increases [14]. These methods differ in their approaches to environmental selection, mating, and diversity preservation, but all require a representation of the solution set rather than a single best point. Bio-inspired algorithm taxonomies distinguish swarm-based, evolutionary, and other behavioral families [4]; [5]. Hybrid and multi-objective engineering studies further show that the value of an added mechanism depends on how it changes convergence, diversity, constraint treatment, and computational cost [6]. In an ant-based method, the central methodological question is therefore not whether pheromone can be used, but how to credit partial decisions when the output is a Pareto archive.

2.2. U-Turn Partial Guidance

The original U-TACO formulation uses U-ants as a prefix mechanism. U-ants visit a limited subset of cities and update the pheromone, while ordinary ants complete the entire tour [1]; [2]. This separation is retained here. A partial continuous vector is never ranked as a valid archive member. Researchers first complete, decode, and evaluate it. If the completed vector enters the archive, pheromone is deposited only on the bins selected by the U-ant. Complete ants, by contrast, can reinforce all coordinates. This credit rule differs from ordinary coordinate-wise perturbation. The full objective vector always generates the quality signal. Thus, a coordinate receives credit only when it participates in a complete non-dominated candidate. The method aims to reduce the risk of rewarding a locally attractive coordinate that performs poorly when combined with the remaining variables.

2.3. Benchmark Families and Indicators

The ZDT suite isolates two-objective problems that include nonconvexity, disconnected fronts, multimodality, and biased solution densities [8]. DTLZ provides scalable objective and variable dimensions and includes linear, concave, multimodal, degenerate, and disconnected structures [9]. WFG uses transformation functions to combine bias, deception, multimodality, nonseparability, degeneracy, and different front shapes [10]. These families remain standard components of multi-objective software frameworks [22]; [23]. No single indicator fully describes a finite approximation set. Hypervolume measures the dominated region in objective space [15]. IGD+ measures the distance from reference points to an approximation using a

weakly Pareto-compliant distance [16]; [17]. The additive epsilon reports the minimum uniform translation required for one set to weakly dominate a reference set. The present study combines these measures and reports archive size, evaluations, and runtime separately.

3. Multi-Objective Optimization Model

A bounded minimization problem is written as:

$$\min_{\mathbf{x} \in \Omega} \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_M(\mathbf{x})]^T, \quad \Omega = \prod_{d=1}^D [l_d, u_d]. \quad (1)$$

A vector $\mathbf{x}^{(a)}$ dominates $\mathbf{x}^{(b)}$ when:

$$\mathbf{x}^{(a)} < \mathbf{x}^{(b)} \Leftrightarrow \left(\forall m: f_m(\mathbf{x}^{(a)}) \leq f_m(\mathbf{x}^{(b)}) \right) \wedge \left(\exists m: f_m(\mathbf{x}^{(a)}) < f_m(\mathbf{x}^{(b)}) \right) \quad (2)$$

The non-dominated archive $\mathcal{A}$ contains candidates for which no other retained candidate satisfies (2). Search takes place in normalized coordinates:

$$z_d = \frac{x_d - l_d}{u_d - l_d}, \quad x_d = l_d + z_d(u_d - l_d), \quad 0 \leq z_d \leq 1 \quad (3)$$

Here, z_d is the normalized coordinate, while l_d and u_d are the lower and upper bounds of the physical variable.

3.1. Continuous MO-U-TACO

3.1.1. Interval Pheromone and Archive-Guided Sampling

Divide each normalized coordinate into B intervals. Let $\tau_{d,b}$ be the pheromone assigned to interval b of coordinate d . Given an archive leader $\mathbf{u}^L$, the engineering-free heuristic used in this benchmark study is a Gaussian proximity term:

$$\eta_{d,b} = 1 + \lambda \exp \left[-\frac{(c_b - u_d^L)^2}{2\sigma_t^2} \right] \quad (4)$$

Where c_b is the interval center. The interval-selection probability is:

$$p_{d,b} = \frac{\tau_{d,b}^\alpha \eta_{d,b}^\beta}{\sum_{q=1}^B \tau_{d,q}^\alpha \eta_{d,q}^\beta} \quad (5)$$

After selecting an interval, sample the coordinate uniformly within it. The sampling width decreases linearly:

$$\sigma_t = \sigma_0 \left(1 - \frac{t-1}{T-1} \right) + \sigma_1 \frac{t-1}{T-1} \quad (6)$$

3.1.2. U-Ant Partial-Decision Phase

A U-ant selects a subset $S \subseteq \{1, \dots, D\}$. Coordinates in S are sampled by (5). Coordinates outside S are completed around a selected archive leader or by bounded random completion. The completed vector is evaluated on all objectives and submitted to the archive. A binary mask records the original U-ant coordinates:

$$m_d = \begin{cases} 1, & d \in S, \\ 0, & d \notin S. \end{cases} \quad (7)$$

If the completed solution is admitted to the archive, the partial pheromone deposit is:

$$\Delta \tau_{d,b}^u = m_d q_u \left(1 + \frac{CD(\mathbf{f})}{CD_{\max}} \right) \quad (8)$$

Where CD denotes crowding distance; equation (8) returns credit only to the sampled coordinates, although the acceptance decision is based on the complete objective vector.

3.2. Complete-Colony Phase and Archive Maintenance

Complete ants sample every coordinate by (5). They deposit in all selected bins. The archive is updated by:

$$\mathcal{A}_{t+1} = \text{Truncate}_K(\text{ND}(\mathcal{A}_t \cup \mathcal{C}_t)) \quad (9)$$

Where ND removes dominated points, and K is the archive capacity. When the archive exceeds K , points with smaller crowding distance are removed. For objective m , the normalized crowding contribution is:

$$CD_i^{(m)} = \frac{f_m^{(i+1)} - f_m^{(i-1)}}{f_m^{\max} - f_m^{\min}} \quad (10)$$

With boundary points assigned infinite distance. Sparse local refinement perturbs a subset of coordinates around diverse archive members. It is accepted only through the same archive update in (9). Pheromone evaporation and deposits are combined as:

$$\tau_{d,b}^{t+1} = (1 - \rho)\tau_{d,b}^t + \Delta\tau_{d,b}^U + \Delta\tau_{d,b}^C + \Delta\tau_{d,b}^E + \Delta\tau_{d,b}^G \quad (11)$$

The last two terms represent elite archive members and a global archive deposit. Pheromone values are bounded numerically to prevent overflow or extinction. Figure 1 summarises the complete sequence of partial guidance, complete-colony construction, archive maintenance, and pheromone updating.

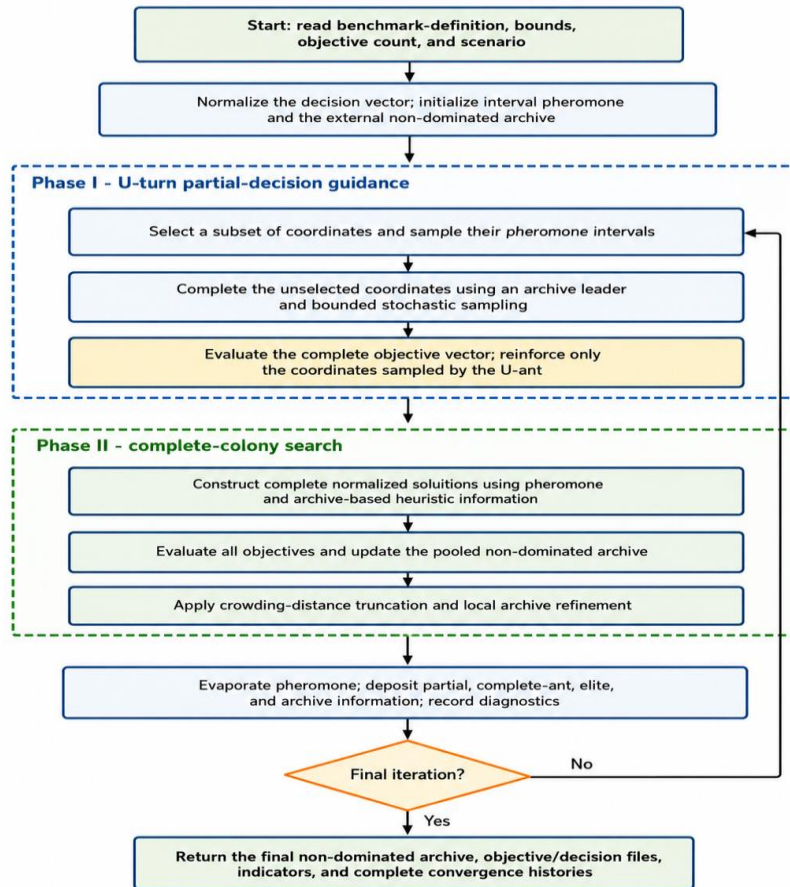


Figure. Continuous multi-objective U-IACO procedure used in the benchmark study.

Figure 1: Continuous MO-U-TACO procedure

4. Benchmark Suite and Experimental Design

4.1. Benchmark Definitions

The main experiment contains five continuous ZDT problems, seven three-objective DTLZ problems, and nine three-objective WFG problems. Researchers exclude ZDT5 because it uses a binary representation. The WFG problems use three objectives, four position variables, and ten distance variables, for a total of 14 decision variables. Table 1 consolidates the complete mathematical definitions, variable domains, dimensional settings, and principal search characteristics of all 21 benchmarks. The ZDT rows explicitly state the two-objective functions; the DTLZ rows use the general multi-objective notation with three objectives in the experiments; and the WFG portion provides the shared scalar transformations, reduction operators, shape functions, and the problem-specific transformation chains for each problem.

Table 1: Mathematical definitions, domains, and search characteristics of the complete benchmark suite

Benchmark	Objective and Auxiliary Equations	Domain and Dimensions	Principal Characteristic
ZDT1	$f_1 = x_1$ $g = 1 + \frac{9}{n-1} \sum_{i=2}^n x_i$ $f_2 = g(1 - \sqrt{f_1/g})$	n=30; xi in [0,1]	Convex Pareto front
ZDT2	$f_1 = x_1$ $g = 1 + \frac{9}{n-1} \sum_{i=2}^n x_i$ $f_2 = g[1 - (f_1/g)^2]$	n=30; xi in [0,1]	Nonconvex Pareto front
ZDT3	$f_1 = x_1$ $g = 1 + \frac{9}{n-1} \sum_{i=2}^n x_i$ $h = 1 - \sqrt{f_1/g} - (f_1/g)\sin(10\pi f_1)$ $f_2 = gh$	n=30; xi in [0,1]	Disconnected Pareto front
ZDT4	$f_1 = x_1$ $q_i = x_i^2 - 10\cos(4\pi x_i)$ $g = 1 + 10(n-1) + \sum_{i=2}^n q_i$ $f_2 = g(1 - \sqrt{f_1/g})$	n=10; x1 in [0,1]; xi in [-5,5] for i>=2	Multimodal decision space
ZDT6	$f_1 = 1 - e^{-4x_1} \sin^6(6\pi x_1)$ $\bar{x} = \frac{1}{n-1} \sum_{i=2}^n x_i$ $g = 1 + 9\bar{x}^{1/4}$ $f_2 = g[1 - (f_1/g)^2]$	n=10; xi in [0,1]	Biased density and nonuniform search
DTLZ1	$k = n - 2$ $g = 100 \left[k + \sum_{i=n-k+1}^n \left((x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5)) \right) \right]$ $f_1 = 1/2 (1 + g)x_1x_2$ $f_2 = 1/2 (1 + g)x_1(1 - x_2)$ $f_3 = 1/2 (1 + g)(1 - x_1)$	M=3; n=7; xi in [0,1]	Linear front; many local fronts
DTLZ2	$k = n - 2, \quad g = \sum_{i=n-k+1}^n (x_i - 0.5)^2$ $f_1 = (1 + g)\cos(\pi x_1/2)\cos(\pi x_2/2)$ $f_2 = (1 + g)\cos(\pi x_1/2)\sin(\pi x_2/2)$	M=3; n=12; xi in [0,1]	Concave spherical front

	$f_3 = (1 + g)\sin(\pi x_1/2)$		
DTLZ3	$k = n - 2$ $g = 100 \left[k + \sum_{i=n-k+1}^n \left((x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5)) \right) \right]$ $f_1 = (1 + g)\cos(\pi x_1/2)\cos(\pi x_2/2)$ $f_2 = (1 + g)\cos(\pi x_1/2)\sin(\pi x_2/2)$ $f_3 = (1 + g)\sin(\pi x_1/2)$	M=3; n=12; xi in [0,1]	Concave front; many local fronts
DTLZ4	$k = n - 2, \quad g = \sum_{i=n-k+1}^n (x_i - 0.5)^2$ $a_1 = \pi x_1^{100}/2, \quad a_2 = \pi x_2^{100}/2$ $f_1 = (1 + g)\cos a_1 \cos a_2$ $f_2 = (1 + g)\cos a_1 \sin a_2$ $f_3 = (1 + g)\sin a_1$	M=3; n=12; xi in [0,1]	Biased mapping
DTLZ5	$k = n - 2, \quad g = \sum_{i=n-k+1}^n (x_i - 0.5)^2$ $\theta_1 = \pi x_1/2$ $\theta_2 = \frac{\pi}{4(1 + g)}(1 + 2gx_2)$ $f_1 = (1 + g)\cos\theta_1\cos\theta_2$ $f_2 = (1 + g)\cos\theta_1\sin\theta_2$ $f_3 = (1 + g)\sin\theta_1$	M=3; n=12; xi in [0,1]	Degenerate Pareto front
DTLZ6	$k = n - 2, \quad g = \sum_{i=n-k+1}^n x_i^{0.1}$ $\theta_1 = \pi x_1/2$ $\theta_2 = \frac{\pi}{4(1 + g)}(1 + 2gx_2)$ $f_1 = (1 + g)\cos\theta_1\cos\theta_2$ $f_2 = (1 + g)\cos\theta_1\sin\theta_2$ $f_3 = (1 + g)\sin\theta_1$	M=3; n=12; xi in [0,1]	Degenerate front with nonuniform mapping
DTLZ7	$f_1 = x_1, \quad f_2 = x_2$ $k = n - 2, \quad g = 1 + \frac{9}{k} \sum_{i=n-k+1}^n x_i$ $h = 3 - \sum_{m=1}^2 \frac{f_m}{1 + g} [1 + \sin(3\pi f_m)]$ $f_3 = (1 + g)h$	M=3; n=22; xi in [0,1]	Disconnected Pareto front
WFG common model	$y_i = z_i/(2i)$ $x_1 = \max(t_3, A_1)(t_1 - 0.5) + 0.5$ $x_2 = \max(t_3, A_2)(t_2 - 0.5) + 0.5$ $x_3 = t_3$ $f_1 = x_3 + 2h_1, \quad f_2 = x_3 + 4h_2$ $f_3 = x_3 + 6h_3$	M=3; k=4; l=10; n=14; zi in [0,2i]	Shared normalization, mapping, and objective construction
WFG scalar transformations	$s_{\text{linear}}(y, A) = \frac{ y - A }{ A - y + A }$ $B_1 = \min(0, y - B) \frac{A(B - y)}{B}$ $B_2 = \min(0, C - y) \frac{(1 - A)(y - C)}{1 - C}$ $b_{\text{flat}}(y, A, B, C) = A + B_1 - B_2$ $b_{\text{poly}}(y, \alpha) = y^\alpha$	Applies to WFG1-WFG9	Bias, flatness, deception, and multimodality operators

	$D_1 = \frac{ y - A + B \left(1 - C + \frac{A - B}{B}\right)}{A - B}$ $D_2 = \frac{ A + B - y \left(1 - C + \frac{1 - A - B}{B}\right)}{1 - A - B}$ $S_{\text{decept}} = 1 + (y - A - B)(D_1 + D_2 + 1/B)$ $\vartheta = \frac{ y - C }{2(C - y + C)}$ $S_{\text{multi}} = \frac{1 + \cos((4A + 2)\pi(1/2 - \vartheta)) + 4B\vartheta^2}{B + 2}$ $v = A - (1 - 2u) 1/2 - u + A $ $b_{\text{param}}(y, u, A, B, C) = y^{B+(C-B)v}$		
WFG reductions and shapes	$r_{\text{sum}}(\mathbf{y}, \mathbf{w}) = \frac{\sum_i w_i y_i}{\sum_i w_i}$ $N = \sum_{j=1}^n \left[y_j + \sum_{r=0}^{A-2} y_j - y_{1+(j+r) \bmod n} \right]$ $c = \text{ceil}(A/2)$ $D = (n/A)c(1 + 2A - 2c)$ $r_{\text{nonsep}}(\mathbf{y}, A) = N/D$ $I_1 = \{1, 2\}, \quad I_2 = \{3, 4\}, \quad I_3 = \{5, \dots, 14\}$ $h_1^{\text{linear}} = x_1 x_2, \quad h_2^{\text{linear}} = x_1(1 - x_2)$ $h_3^{\text{linear}} = 1 - x_1$ $h_1^{\text{convex}} = (1 - \cos \pi x_1/2)(1 - \cos \pi x_2/2)$ $h_2^{\text{convex}} = (1 - \cos \pi x_1/2)(1 - \sin \pi x_2/2)$ $h_3^{\text{convex}} = 1 - \sin(\pi x_1/2)$ $h_1^{\text{concave}} = \sin(\pi x_1/2)\sin(\pi x_2/2)$ $h_2^{\text{concave}} = \sin(\pi x_1/2)\cos(\pi x_2/2)$ $h_3^{\text{concave}} = \cos(\pi x_1/2)$ $h_3^{\text{mixed}} = \left[1 - x_1 - \frac{\cos(2A\pi x_1 + \pi/2)}{2A\pi} \right]^\alpha$ $h_3^{\text{disc}} = 1 - x_1^\alpha \cos^2(A\pi x_1^\beta)$	WFG group reductions and M=3 shape functions	Nonseparability and Pareto-front geometry
WFG1	$t_i^{(1)} = y_i \quad (i \leq k)$ $t_i^{(1)} = s_{\text{linear}}(y_i, 0.35) \quad (i > k)$ $t_i^{(2)} = t_i^{(1)} \quad (i \leq k)$ $t_i^{(2)} = b_{\text{flat}}(t_i^{(1)}, 0.8, 0.75, 0.85) \quad (i > k)$ $t_i^{(3)} = b_{\text{poly}}(t_i^{(2)}, 0.02)$ $t_m^{(4)} = r_{\text{sum}}(\mathbf{t}_{I_m}^{(3)}, \{2i: i \in I_m\})$ $h_1, h_2 = h^{\text{convex}}, \quad h_3 = h^{\text{mixed}}(5, 1)$	zi in [0,2i]; M=3; k=4; l=10	Mixed convex/concave front; biased and flat regions
WFG2	$t_i^{(1)} = y_i \quad (i \leq k)$ $t_i^{(1)} = s_{\text{linear}}(y_i, 0.35) \quad (i > k)$ $t_i^{(2)} = t_i^{(1)} \quad (i \leq k)$ $t_{k+j}^{(2)} = r_{\text{nonsep}}\left((t_{k+2j-1}^{(1)}, t_{k+2j}^{(1)}), 2\right)$ $t_m^{(3)} = r_{\text{sum}}(\mathbf{t}_{I_m}^{(2)}, \mathbf{1})$ $h_1, h_2 = h^{\text{convex}}, \quad h_3 = h^{\text{disc}}(5, 1, 1)$	j=1,...,l/2; zi in [0,2i]	Nonseparable and disconnected
WFG3	$t_i^{(1)} = y_i \quad (i \leq k)$ $t_i^{(1)} = s_{\text{linear}}(y_i, 0.35) \quad (i > k)$ $t_{k+j}^{(2)} = r_{\text{nonsep}}\left((t_{k+2j-1}^{(1)}, t_{k+2j}^{(1)}), 2\right)$ $t_m^{(3)} = r_{\text{sum}}(\mathbf{t}_{I_m}^{(2)}, \mathbf{1})$ $\mathbf{A} = (1, 0), \quad h_1, h_2, h_3 = h^{\text{linear}}$	j=1,...,l/2; zi in [0,2i]	Degenerate linear front

WFG4	$t_i^{(1)} = s_{\text{multi}}(y_i, 30, 10, 0.35)$ $t_m^{(2)} = r_{\text{sum}}(\mathbf{t}_{l_m}^{(1)}, \mathbf{1})$ $h_1, h_2, h_3 = h^{\text{concave}}$	z_i in $[0, 2i]$; $M=3$; $k=4$; $l=10$	Multimodal
WFG5	$t_i^{(1)} = s_{\text{decept}}(y_i, 0.35, 0.001, 0.05)$ $t_m^{(2)} = r_{\text{sum}}(\mathbf{t}_{l_m}^{(1)}, \mathbf{1})$ $h_1, h_2, h_3 = h^{\text{concave}}$	z_i in $[0, 2i]$; $M=3$; $k=4$; $l=10$	Deceptive
WFG6	$t_i^{(1)} = y_i \quad (i \leq k)$ $t_i^{(1)} = s_{\text{linear}}(y_i, 0.35) \quad (i > k)$ $t_m^{(2)} = r_{\text{nonsep}}(\mathbf{t}_{l_m}^{(1)}, l_m)$ $h_1, h_2, h_3 = h^{\text{concave}}$	z_i in $[0, 2i]$; $M=3$; $k=4$; $l=10$	Nonseparable
WFG7	$u_i = r_{\text{sum}}((y_{i+1}, \dots, y_n), \mathbf{1})$ $t_i^{(1)} = b_{\text{param}}(y_i, u_i, A_p, B_p, C_p) \quad (i \leq k)$ $t_i^{(1)} = y_i \quad (i > k)$ $t_i^{(2)} = t_i^{(1)} \quad (i \leq k)$ $t_i^{(2)} = s_{\text{linear}}(t_i^{(1)}, 0.35) \quad (i > k)$ $t_m^{(3)} = r_{\text{sum}}(\mathbf{t}_{l_m}^{(2)}, \mathbf{1})$ $A_p = 0.98/49.98, \quad B_p = 0.02, \quad C_p = 50$	z_i in $[0, 2i]$; $M=3$; $k=4$; $l=10$	Parameter-dependent bias
WFG8	$u_i = r_{\text{sum}}((y_1, \dots, y_{i-1}), \mathbf{1})$ $t_i^{(1)} = b_{\text{param}}(y_i, u_i, A_p, B_p, C_p) \quad (i > k)$ $t_i^{(1)} = y_i \quad (i \leq k)$ $t_i^{(2)} = t_i^{(1)} \quad (i \leq k)$ $t_i^{(2)} = s_{\text{linear}}(t_i^{(1)}, 0.35) \quad (i > k)$ $t_m^{(3)} = r_{\text{sum}}(\mathbf{t}_{l_m}^{(2)}, \mathbf{1})$	z_i in $[0, 2i]$; $M=3$; $k=4$; $l=10$	Nonseparable parameter dependence

4.2. Parameter Scenarios

The scenarios vary colony size, U-ant count, iteration budget, archive capacity, evaporation, partial fraction, and local refinement. They are controlled alternatives, not universal parameter recommendations. Table 2 lists the numerical settings used for S1-S8.

Table 2: Main parameter settings

Set	Description	Ants	U-ants	Iter.	Bins	Archive	Evap.	Partial fraction	Local trials
S1	Conservative	8	4	10	15	30	0.15	0.5	0
S2	Balanced	10	5	12	20	40	0.20	0.5	1
S3	Exploratory	12	7	12	20	40	0.30	0.67	0
S4	Exploitative	10	5	15	25	40	0.12	0.5	1
S5	Rapid evaporation	10	5	12	20	40	0.45	0.5	1
S6	Strong U-turn	10	8	12	20	40	0.20	0.67	1
S7	Large colony	16	8	12	25	50	0.20	0.5	1
S8	Precision archive search	12	6	18	30	60	0.15	0.5	2

4.3. Repeated-Run Protocol

Researchers executed each scenario with seeds 1–50 on every problem. The total number of independent problem runs was:

$$21 \text{ problems} \times 8 \text{ scenarios} \times 50 \text{ seeds} = 8400 \quad (12)$$

The 10-run sample contains seeds 1–10, the 30-run sample contains seeds 1–30, and the 50-run sample contains seeds 1–50. This nesting prevents changes in seed composition from being confused with changes in sample size. Because researchers did not sample the exact analytical front for every WFG transformation, each problem used a pooled empirical reference set formed from the non-dominated union of all final archives. Researchers normalized the pooled ideal and nadir values to objective

values before calculating indicators. This convention supports within-study scenario comparisons but must not be interpreted as an absolute distance to the theoretical Pareto front. For approximation set A and reference set R , IGD⁺ is:

$$IGD^+(A, R) = \frac{1}{|R|} \sum_{r \in R} \min_{a \in A} \sqrt{\sum_{m=1}^M \max(a_m - r_m, 0)^2} \quad (13)$$

The normalized hypervolume ratio is:

$$HVR(A) = \frac{HV(A; \mathbf{r})}{HV(R; \mathbf{r})} \quad (14)$$

Using $\mathbf{r} = (1.1, \dots, 1.1)$ after normalization. Lower IGD⁺ and epsilon values are preferred; larger HVR is preferred. A per-problem composite rank averages the IGD⁺, HVR, and epsilon ranks. Researchers applied Friedman's test to matched seed-level aggregate scores [18]; [21]. Kendall's coefficient was calculated as:

$$W = \frac{\chi_F^2}{N(k-1)} \quad (15)$$

Where N is the number of matched seeds and $k = 8$ scenarios; the best aggregate scenario was compared with each alternative using paired Wilcoxon signed-rank tests [20], followed by Holm correction [19].

5. Results

5.1. Aggregate Performance

S7 achieved the best mean composite rank, mean IGD⁺ rank, and mean hypervolume rank.

Table 3: Fifty-run aggregate results

Set	Composite rank	IGD ⁺ rank	HV rank	Best problems	Mean IGD ⁺	Mean HV ratio	Mean epsilon	Runtime (s)	Evaluations
S7	1.381	1.19	1.571	12	0.479148	0.579963	0.665521	0.1327	350
S8	2.825	3.095	2.905	3	0.473454	0.527794	0.663689	0.1519	419.8
S6	3.349	3.19	3.571	0	0.583993	0.501393	0.77892	0.0909	268
S3	4.159	4.095	4.286	0	0.651002	0.494102	0.844536	0.1104	268
S5	4.794	4.714	4.524	0	0.633234	0.485637	0.836437	0.0748	232
S2	5.429	5.429	5.381	0	0.639718	0.474997	0.83722	0.0776	232
S4	6.46	6.619	6.333	0	0.673576	0.445789	0.879317	0.1087	280
S1	7.603	7.667	7.429	0	0.778854	0.401865	0.987161	0.0498	150

It ranked first on 12 problems. S8 ranked second and was first on three problems.

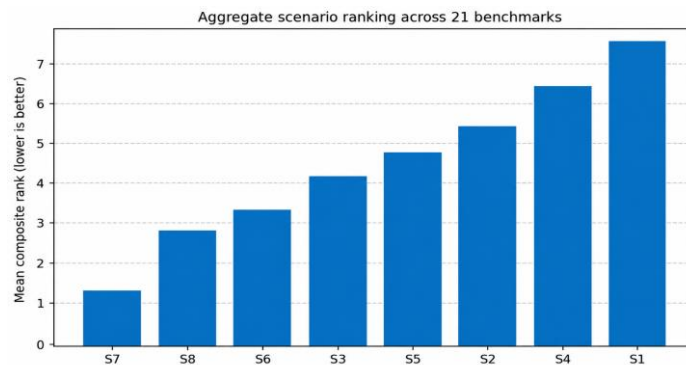


Figure 2: Aggregate scenario ranks across the 21 benchmarks

The arithmetic mean of problem-level IGD+ was marginally smaller for S8 than for S7, but S7 obtained the larger hypervolume ratio and a much stronger problem-wise rank profile. This difference illustrates why a single pooled arithmetic mean should not replace per-problem ranks or multiple indicators. Table 3 reports the complete aggregate statistics, and Figure 2 visualizes the resulting scenario-rank ordering. The distribution plots in Figures 3 and 4 show the same ordering from two perspectives.

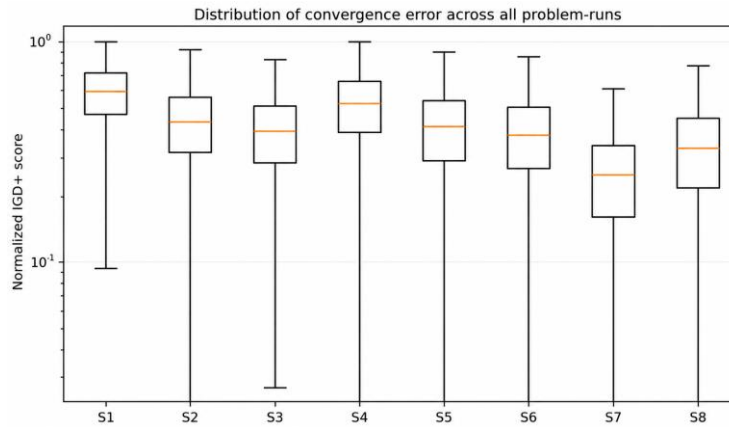


Figure 3: Distribution of normalized IGD+ scores

S7 has the strongest normalized IGD+ distribution and the highest hypervolume concentration. S8 remains competitive but uses a larger archive and evaluation budget.

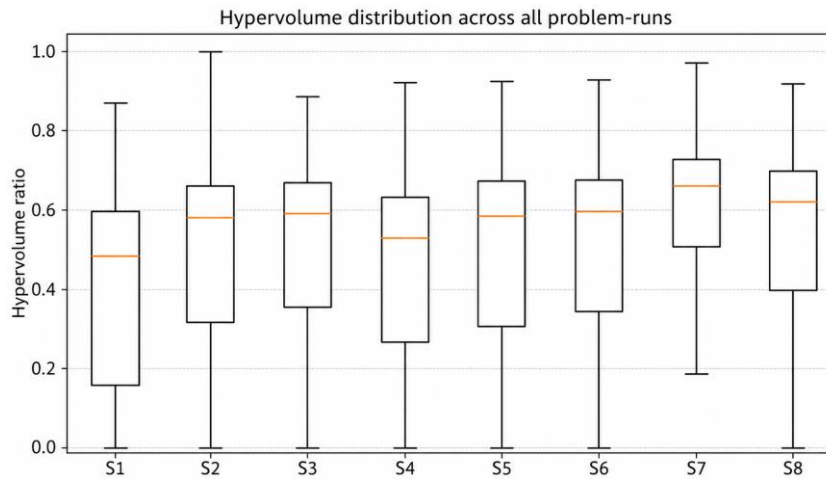


Figure 4: Distribution of normalized hypervolume ratios

5.2. Benchmark-Family Behavior

S7 ranked first in all three families. Its strongest mean rank occurred on DTLZ, followed by WFG and ZDT. S8 was especially competitive on WFG and obtained the best setting on WFG1, WFG5, and WFG6, but its ZDT rank was weaker.

Table 4: Mean composite rank by benchmark family

Set	DTLZ rank	WFG rank	ZDT rank
S1	7.333	7.926	7.4
S2	5.571	5.704	4.733
S3	3.619	4.852	3.667
S4	6.286	6.556	6.533
S5	4.952	4.704	4.733
S6	4.095	3.037	2.867

S7	1.238	1.37	1.6
S8	2.905	1.852	4.467

S6 provided the third-best aggregate result and was comparatively stronger on ZDT and WFG than on DTLZ.

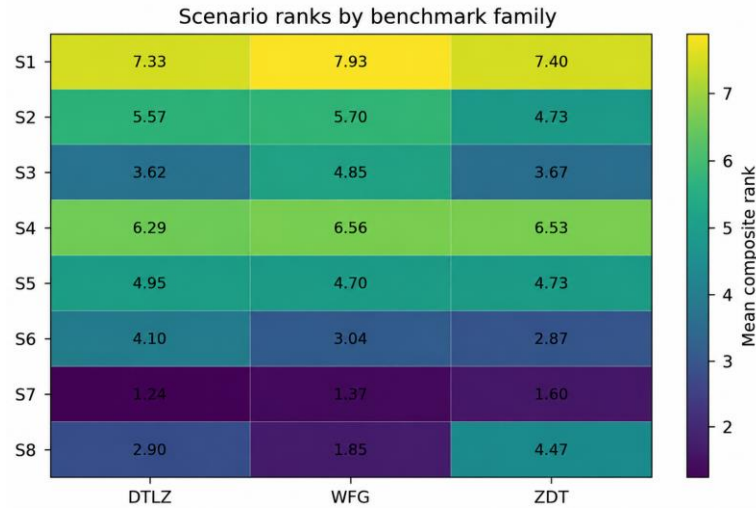


Figure 5: Scenario ranks by benchmark family

Table 4 reports the family-level values and Figure 5 visualizes them, while Table 5 identifies the best-ranked scenario for each benchmark.

Table 5: Best-ranked scenario for each benchmark

Problem	Family	Best Set	Composite Rank	IGD+ Rank	HV Rank	Epsilon Rank
DTLZ1	DTLZ	S8	1.333	2	1	1
DTLZ2	DTLZ	S7	1	1	1	1
DTLZ3	DTLZ	S7	1	1	1	1
DTLZ4	DTLZ	S7	1	1	1	1
DTLZ5	DTLZ	S7	1	1	1	1
DTLZ6	DTLZ	S8	1	1	1	1
DTLZ7	DTLZ	S7	1	1	1	1
WFG1	WFG	S8	1	1	1	1
WFG2	WFG	S7	1	1	1	1
WFG3	WFG	S7	1	1	1	1
WFG4	WFG	S7	1.333	1	1	2
WFG5	WFG	S8	1.333	2	1	1
WFG6	WFG	S8	1	1	1	1
WFG7	WFG	S7	1	1	1	1
WFG8	WFG	S7	1	1	1	1
WFG9	WFG	S7	1.333	1	1	2
ZDT1	ZDT	S7	1	1	1	1
ZDT2	ZDT	S7	2.167	1	4.5	1
ZDT3	ZDT	S7	1	1	1	1
ZDT4	ZDT	S8	2.167	1	4.5	1
ZDT6	ZDT	S7	1	1	1	1

5.3. Statistical Comparison

Researchers rejected the null hypothesis of equal matched-score distributions for all sample sizes. Kendall's remained close to 0.81, indicating that the scenario effect was large relative to within-seed variation under the adopted aggregate score. Table 6 provides the complete Friedman statistics. S7 was favored over every alternative after correction.

Table 6: Friedman tests for the nested samples

Runs	Metric	Friedman statistic	p-value	Kendall W
10	Median normalized IGD+ across 21 problems	58	3.781e-10	0.828571
30	Median normalized IGD+ across 21 problems	171.122	1.456e-33	0.814868
50	Median normalized IGD+ across 21 problems	284.227	1.408e-57	0.812076

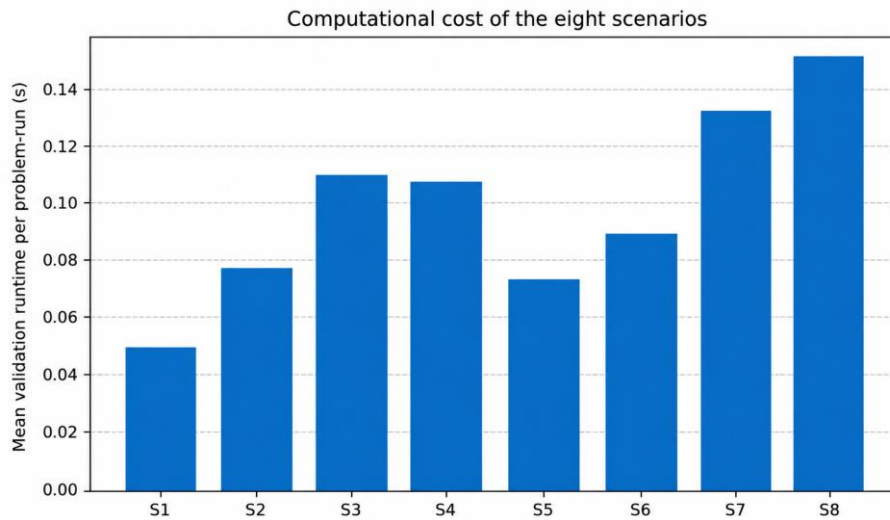
The smallest median advantage was against S8, but this comparison remained significant, with a rank-biserial effect size of 0.88. The result concerns the normalized aggregate score defined in the protocol; it does not imply that S7 has lower IGD+ on every individual problem. Table 7 reports the corresponding Wilcoxon statistics, Holm-adjusted probabilities, and effect sizes.

Table 7: Holm-adjusted comparisons for the 50-run sample

Reference	Comparison	W	Raw p	Holm p	Rank-Biserial	Median Advantage	Significant
S7	S1	0	8.882e-16	6.217e-15	1	0.345821	True
S7	S2	0	8.882e-16	6.217e-15	1	0.181260	True
S7	S3	3	4.441e-15	8.882e-15	0.92	0.139825	True
S7	S4	0	8.882e-16	6.217e-15	1	0.271713	True
S7	S5	0	8.882e-16	6.217e-15	1	0.165624	True
S7	S6	0	8.882e-16	6.217e-15	1	0.122218	True
S7	S8	9	2.931e-14	2.931e-14	0.88	0.082633	True

5.4. Computational Effort

S1 used the smallest budget, with 150 evaluations per problem run and a mean validation runtime of 0.0498 s. S7 used 350 evaluations and 0.1327 s. S8 used approximately 419.8 evaluations and 0.1519s.

**Figure 6:** Mean validation runtime by scenario

The larger S7 colony improved archive coverage more consistently than the precision-oriented S8 setting under this short iteration budget. Figure 6 shows the runtime increase associated with the larger scenarios. The Python validation implementation in the execution environment measured these times. They are not MATLAB benchmarks. Evaluation counts are the more portable measure of search effort.

5.5. Convergence and Archive Dynamics

The DTLZ2 history shows that S7 and S8 reduce IGD+ faster than S1. S8 continues refining with the largest final budget, whereas S7 reaches a competitive archive using fewer evaluations. Figure 7 shows these convergence patterns. Pheromone entropy decreases as interval probabilities concentrate. On WFG4, S8 produces the strongest concentration, consistent with its smaller terminal sampling width. S7 maintains more diversity through its larger colony and archive.

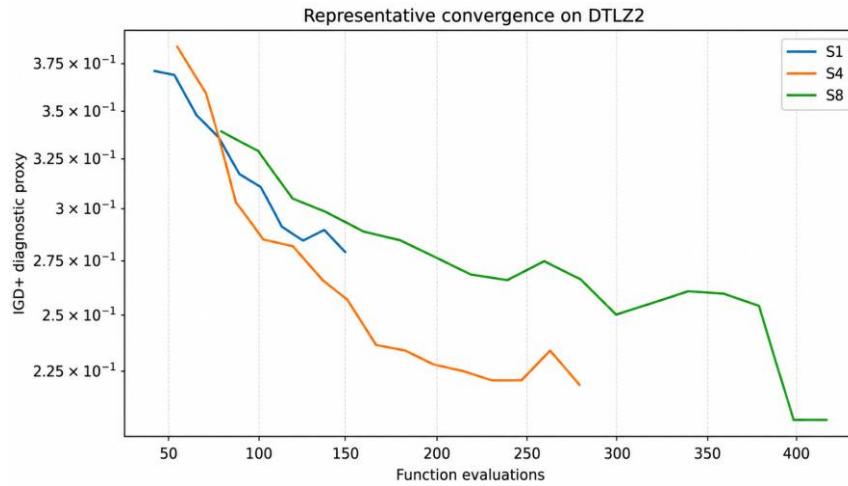


Figure 7: Representative IGD+ convergence on DTLZ2

Figure 8 records the corresponding entropy histories.

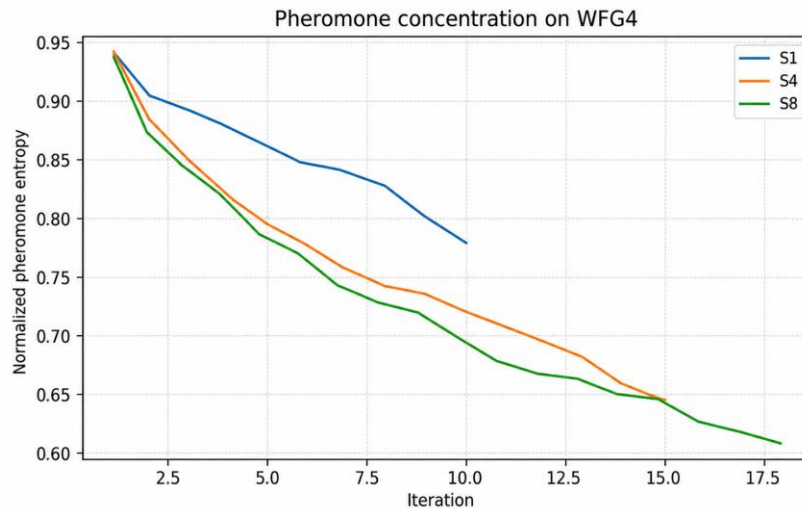


Figure 8: Pheromone entropy on WFG4

The representative fronts show that the method can approximate convex, disconnected, concave, and multimodal structures.

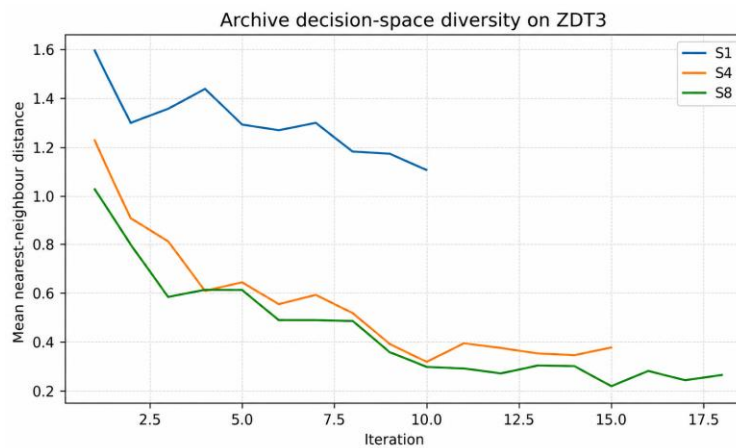


Figure 9: Decision-space archive diversity on ZDT3

The pooled empirical set is plotted only as an internal reference, not as proof that the analytical front has been recovered. Figure 9 shows decision-space archive diversity on ZDT3, whereas Figures 10 to 13 present representative objective-space approximations for ZDT1, ZDT3, DTLZ2, and WFG4, respectively.

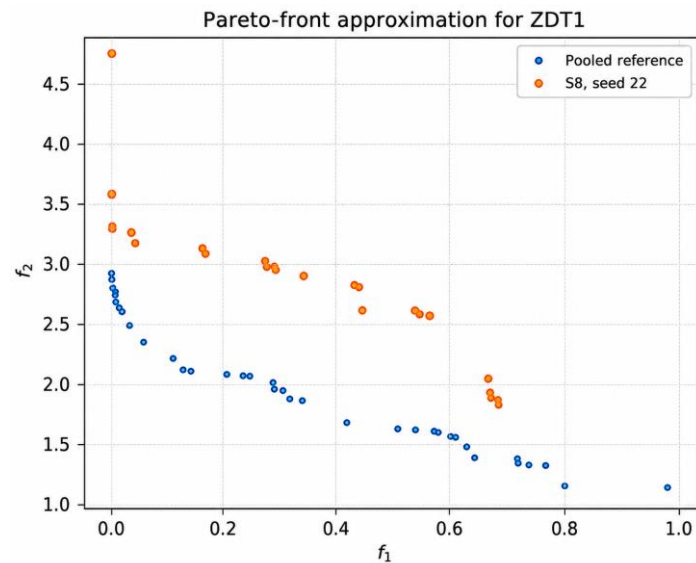


Figure 10: Representative S8 front on ZDT1

Figure 11 depicts the Pareto-front approximation suitable for the ZDT3 benchmark challenge. This approximation includes results from seed 22 and the pooled reference solutions. The orange dots are a representation of the solutions obtained by S8, and the blue dots are a representation of the pooled reference. The distribution of points in the objective space f_1 - f_2 not only shows the variances in convergence between the developed solutions and the reference front, but also indicates the discontinuous nature of the ZDT3 Pareto front, which is a characteristic typically observed.

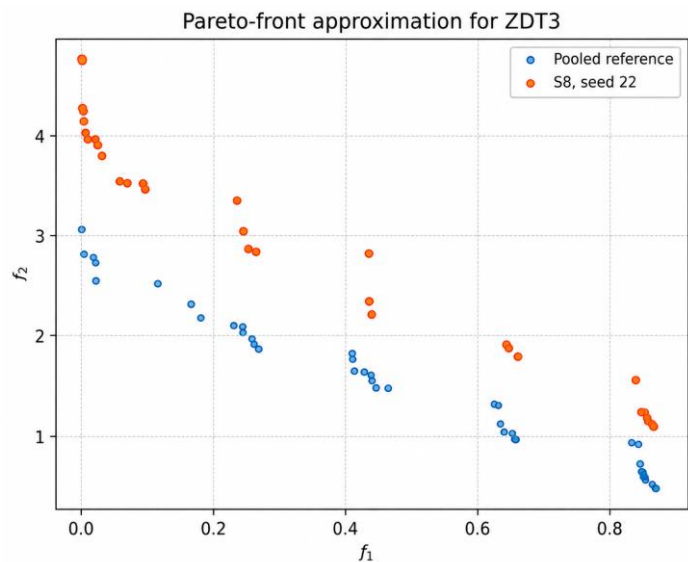


Figure 11: Representative S8 front on ZDT3

Figure 12 depicts the approximate representation of the Pareto front that was utilized for the DTLZ2 benchmark problem. There are three objectives in this approximation, designated by the letters f_1 , f_2 , and f_3 in that order. The solutions found using S8 seed 43 are shown as orange points, whereas the pooled reference solutions are shown as blue points. Each set of points represents a different answer. To show the convergence and diversity of solutions in the three-dimensional objective space, the distribution illustrates convergence. Additionally, this distribution makes this style of illustration available to the public.

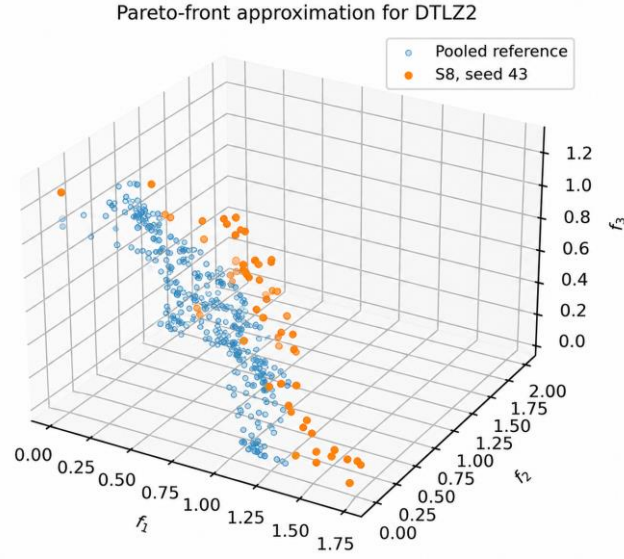


Figure 12: Representative S8 front on DTLZ2

Figure 13 approximates the Pareto front of the WFG4 benchmark problem with three objectives: f_1 , f_2 , and f_3 . Orange: S8 seed 41 solutions, blue: pooled reference. Reasonable S8 convergence and reference front objective space coverage.

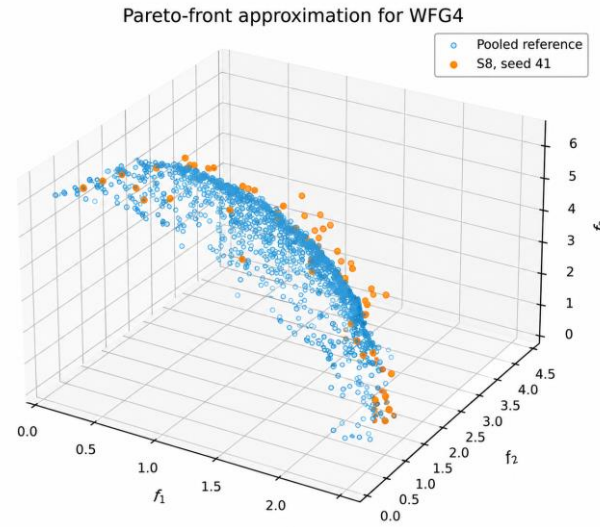


Figure 13: Representative S8 front on WFG4

5.6. Stability with 10, 30, and 50 Runs

S7 was first at each sample size, with mean composite ranks of 1.683, 1.492, and 1.381. S8 remained second at 30 and 50 runs. The increasing separation between S7 and weaker settings indicates that the larger sample mainly improves estimate precision rather than changing the central ordering.

Table 8: Mean composite rank across nested run counts

Set	10 runs	30 runs	50 runs
S1	7.397	7.587	7.603
S2	4.952	5.238	5.429
S3	3.762	4.079	4.159
S4	6.381	6.397	6.46
S5	4.714	4.73	4.794

S6	3.921	3.556	3.349
S7	1.683	1.492	1.381
S8	3.19	2.921	2.825

Table 8 reports the nested-sample ranks, and Figure 14 visualizes their stability across 10, 30, and 50 runs.

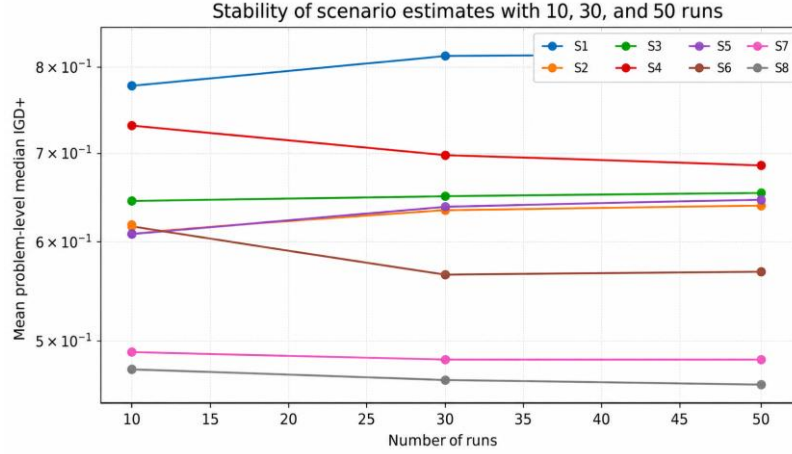


Figure 14: Run-count stability for the eight scenarios

5.7. Discussion

The main result is that the most exploitative configuration did not perform best. S4 used stronger pheromone and heuristic exponents, but its smaller colony and archive produced a mean composite rank of 6.460. S8 used the largest archive, finest interval partition, deepest local refinement, and smallest terminal sampling width. It produced the smallest arithmetic mean IGD+ and epsilon, yet it did not match S7's hypervolume and per-problem consistency. The large-colony scenario therefore provided the most effective balance between sampling coverage and pheromone learning for this budget. The partial U-ant mechanism remained active across all scenarios, but the results do not isolate its causal contribution because the study did not include an ordinary continuous ACO ablation. The study supports parameter selection within MO-U-TACO; it does not prove that partial U-turn guidance outperforms complete-ant search. A subsequent paper should compare complete-ant-only, partial-credit, and random-partial variants under identical evaluation budgets. The empirical reference front is another important limitation. Pooling all final archives avoids using a single scenario as the reference, but it can underestimate the distance to the true front when each configuration misses the same region. Analytical or high-resolution reference fronts should be used before comparing MO-U-TACO with established algorithms. WFG was evaluated with $l = 10$ rather than the frequently used $l = 20$, and the main study used two or three objectives only. The code structure supports additional objectives, but many-objective conclusions require separate experiments.

6. Conclusion

Researchers formulated a continuous multi-objective U-TACO method by preserving the original distinction between partial U-ant guidance and complete-ant solution construction. Partial coordinates receive pheromone credit only after completion, full objective evaluation, and admission to the archive. An external non-dominated archive, crowding-distance truncation, interval pheromone, and sparse local refinement extend the mechanism to bounded real-valued benchmarks. Across 21 ZDT, DTLZ, and WFG problems, eight scenarios, and 50 matched seeds, S7 obtained the best mean composite rank and the largest number of best-ranked problems. S8 provided the closest alternative and the smallest arithmetic mean IGD+, but required more evaluations and produced a lower mean hypervolume ratio. The statistical and nested run-count analyses support S7 as the main configuration for this experimental protocol. Further work should perform a complete ablation, use analytical reference fronts, increase the objective count and evaluation budget, and compare MO-U-TACO with established multi-objective algorithms under identical stopping conditions.

6.1. Reproducibility and Limitations

The accompanying package contains the Python validation implementation, MATLAB translation, eight scenario definitions, all 8,400 run records, objective and decision archives, pooled reference fronts, convergence histories, statistical outputs, and the data for Figures 11 to 14. Seeds are explicit. The MATLAB package writes FUN.csv, VAR.csv, history files, and MAT

checkpoints for each run. The study has five boundaries. It compares parameter scenarios rather than external algorithms. It uses pooled empirical reference fronts. Runtime values are environment-specific Python measurements. WFG uses a reduced distance dimension. Finally, the evaluation budget is intentionally small to make 8,400 runs tractable; larger budgets may alter the relative value of local refinement and archive size.

Acknowledgment: The authors gratefully acknowledge Technical College of Informatics-Akre, Akre University for Applied Sciences, Olabisi Onabanjo University, and Zewail City of Science and Technology for their valuable academic support, guidance, research resources, and institutional assistance throughout this research.

Data Availability Statement: The data for this study can be made available upon request to the corresponding author.

Funding Statement: No external funding was received for this study.

Conflicts of Interest Statement: The authors declare no conflict of interest.

Ethics and Consent Statement: This research adheres to ethical guidelines, obtaining informed consent from all participants.

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